2014 AMC 10B Solutions

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1. Leah has 13 coins, all of which are pennies and nickels. If she had one more nickel than she has now, then she would have the same number of pennies and nickels. In cents, how much are Leah's coins worth?

A 33

B 35

C 37

D 39

E 41

Solution(s):

Let the number of pennies be p. Then, the number of nickels is $13 - p$ and $p - 1$, so $13 - p = p - 1$. This means we have 7 pennies and 6 nickels.

Therefore, the number of cents is $7 + 6 \cdot 5 = 37$.

Thus, the correct answer is **C**.

2. What is $\dfrac{2^3+2^3}{2^{-3}+2^{-3}}$?

A 16

B 24

C 32

D 48

E 64

Solution(s):

$$\begin{aligned}\frac{2^3+2^3}{2^{-3}+2^{-3}} &= \frac{2\cdot 2^3}{2\cdot 2^{-3}}\\ &= 2^6\\ &= 64\end{aligned}$$

Thus, the correct answer is **E**.

3. Randy drove the first third of his trip on a gravel road, the next 20 miles on pavement, and the remaining one-fifth on a dirt road. In miles how long was Randy's trip?

A 30

B $\frac{400}{11}$

C $\frac{75}{2}$

D 40

E $\frac{300}{7}$

Solution(s):

Let the path distance be t. Then, we get:

$$\begin{aligned} t &= \frac{t}{5} + 20 + \frac{t}{3} \\ t &= 20 + \frac{8t}{15} \\ \frac{7}{15}t &= 20 \\ t &= \frac{300}{7}. \end{aligned}$$

Thus, the correct answer is **E**.

4. Susie pays for 4 muffins and 3 bananas. Calvin spends twice as much paying for 2 muffins and 16 bananas. A muffin is how many times as expensive as a banana?

A $\frac{3}{2}$

B $\frac{5}{3}$

C $\frac{7}{4}$

D 2

E $\frac{13}{4}$

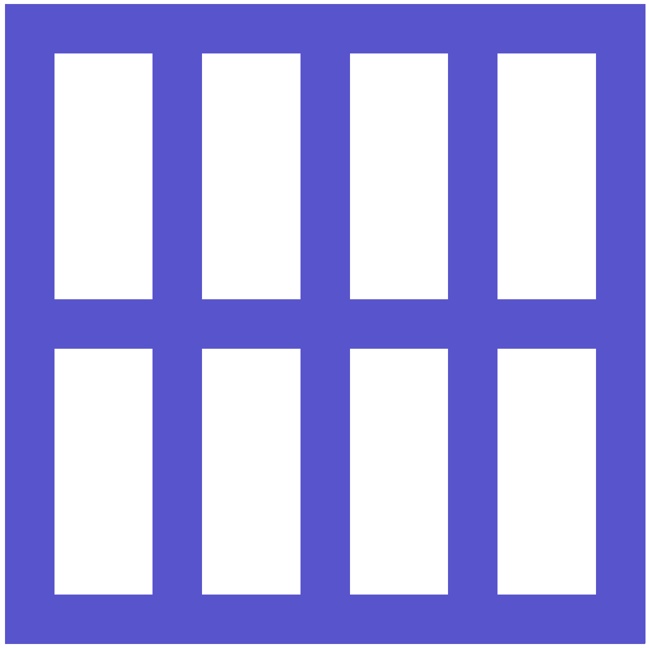
Solution(s):

Let the price for a muffin be m and let the price for a banana be b. Then,

$$\begin{aligned} 2(4m+3b) &= 16b+2m \\ 8m+6b &= 16b+2m \\ 10b &= 6m \\ m &= \frac{5}{3}b. \end{aligned}$$

Thus, the correct answer is **B**.

5. Doug constructs a square window using 8 equal-size panes of glass, as shown. The ratio of the height to width for each pane is $5:2$, and the borders around and between the panes are 2 inches wide. In inches, what is the side length of the square window?



A 26

B 28

C 30

D 32

E 34

Solution(s):

Let the smaller side of a pane be a distance of x.

Then, the side length is $5 \cdot 2 + 4x$. Also, the other direction has pane lengths of $2.5x$.

This means the side length is $3 \cdot 2 + 2(2.5x)$. We solve for x as follows:

$$10 + 4x = 6 + 5x$$
$$x = 4$$

Therefore, the side length is

$$10 + 4 \cdot 4 = 26.$$

Thus, the correct answer is **A**.

6. Orvin went to the store with just enough money to buy 30 balloons. When he arrived, he discovered that the store had a special sale on balloons: buy 1 balloon at the regular price and get a second at $\frac{1}{3}$ off the regular price. What is the greatest number of balloons Orvin could buy?

A 33

B 34

C 36

D 38

E 39

Solution(s):

Suppose we buy 6 balloons. Then, we can buy 3 at full price and 3 at a price of $\frac{2}{3}$ of a ballon.

Therefore, we can buy it at a price of 5 balloons. Thus, with the money to buy 30 balloons, we could buy $30 \cdot \frac{6}{5} = 36$ balloons.

Thus, the correct answer is **C**.

7. Suppose $A > B > 0$ and A is $x\%$ greater than B. What is x?

A $100\left(\frac{A-B}{B}\right)$

B $100\left(\frac{A+B}{B}\right)$

C $100\left(\frac{A+B}{A}\right)$

D $100\left(\frac{A-B}{A}\right)$

E $100\left(\frac{A}{B}\right)$

Solution(s):

By definition, we know

$$\begin{aligned} A &= \frac{x+100}{100}B \\ &= B + \frac{x}{100}B. \end{aligned}$$

This implies,

$$\begin{aligned} (A-B) &= \frac{x}{100}B \\ 100\left(\frac{A-B}{B}\right) &= x. \end{aligned}$$

Thus, the correct answer is **A**.

8. A truck travels $\frac{b}{6}$ feet every t seconds. There are 3 feet in a yard. How many yards does the truck travel in 3 minutes?

A $\frac{b}{1080t}$

B $\frac{30t}{b}$

C $\frac{30b}{t}$

D $\frac{10t}{b}$

E $\frac{10b}{t}$

Solution(s):

This means it travels $\frac{b}{18}$ yards in t seconds since a yard is 3 feet. Then, in one second, it travels $\frac{b}{18t}$ yards.

Therefore, in 3 minutes which is 180 seconds, it travels

$$\frac{b}{18t} \cdot 180 = \frac{10b}{t}.$$

Thus, the correct answer is **E**.

9. For real numbers w and z,

$$\frac{\frac{1}{w}+\frac{1}{z}}{\frac{1}{w}-\frac{1}{z}}=2014.$$

What is $\frac{w+z}{w-z}$?

A -2014

B $\dfrac{-1}{2014}$

C $\dfrac{1}{2014}$

D 1

E 2014

Solution(s):

Observe that:

$$\begin{aligned}\frac{wz}{wz}\cdot\frac{\frac{1}{w}+\frac{1}{z}}{\frac{1}{w}-\frac{1}{z}}&=2014\\ \frac{w+z}{z-w}&=2014\\ \frac{w+z}{w-z}&=-1\cdot 2014\\ \frac{w+z}{w-z}&=-2014.\end{aligned}$$

Thus, the correct answer is **A**.

10. In the addition shown below A, B, C, and D are distinct digits. How many different values are possible for D?

$$\begin{array}{r} ABBCB \\ +\ BCADA \\ \hline DBDDD \end{array}$$

A 2

B 4

C 7

D 8

E 9

Solution(s):

From the leftmost digit, we know $A+B=D\leq 9$ since it has no carryover.

This means there is no carryover when adding the units digit, making $C+D=D$. Thus, $C=0$. Then, since $A,B\neq 0$ and $A\neq B$, we know

$$D=A+B\geq 1+2=3.$$

Since $3\leq D\leq 9$, we have 7 values for D.

Thus, the correct answer is **C**.

11. For the consumer, a single discount of $n\%$ is more advantageous than any of the following discounts:

(1) Two successive 15% discounts.

(2) Three successive 10% discounts.

(3) A 25% discount followed by a 5% discount.

What is the smallest possible positive integer value of n?

A 27

B 28

C 29

D 31

E 33

Solution(s):

We need to find the smallest possible n such that

$$1-\frac{n}{100}<(0.85)^2,$$

$$1-\frac{n}{100}<(0.9)^3,$$

$$1-\frac{n}{100}<(0.75)(0.95).$$

Note that

$$0.75\cdot 0.95=0.85^2-0.1^2<0.85^2,$$

so we don't need to worry about the first condintion since the last condition is true. Then, the second condition yields

$$1-\frac{n}{100}<0.729.$$

$$n>27.1.$$

Also, we also can see that

$$1 - \frac{n}{100} < 0.95 \cdot 0.75$$

$$1 - \frac{n}{100} < \frac{3}{4} \cdot \frac{19}{20}$$

$$1 - \frac{n}{100} < \frac{57}{80}$$

$$1 - \frac{n}{100} < \frac{285}{400}.$$

Therefore,

$$n > \frac{115}{4} = 28.75.$$

Combining our conditions yields a smallest n of $n = 29$.

Thus, the correct answer is **C**.

12. The largest divisor of $2,014,000,000$ is itself. What is its fifth-largest divisor?

A $125,875,000$

B $201,400,000$

C $251,750,000$

D $402,800,000$

E $503,500,000$

Solution(s):

The fifth-largest divisor is $2,014,000,000$ divided by the fifth smallest divisor.

The prime factoriztion of $2,014,000,000$ is:

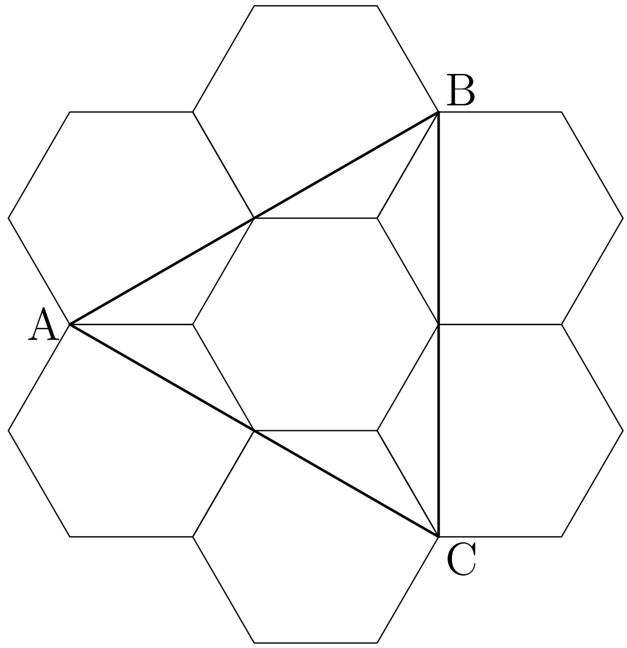
$$2^7 \cdot 5^6 \cdot 19 \cdot 53.$$

This makes the first 5 smallest divisors $1, 2, 4, 5, 8$ Therefore, the fifth smallest divisor is 8, and the fifth largest divisor must be:

$$\frac{2014000000}{8} = 251750000.$$

Thus, the correct answer is **C**.

13. Six regular hexagons surround a regular hexagon of side length 1 as shown. What is the area of $\triangle ABC$?



A $2\sqrt{3}$

B $3\sqrt{3}$

C $1+3\sqrt{2}$

D $2+2\sqrt{3}$

E $3+2\sqrt{3}$

Solution(s):

Since $AB = BC = AC$ by rotational symmetry, we know it is an equilateral triangle.

Then, one-fourth of AB can be found as a the leg of a right triangle with hypotenuse 1 and is opposite to the 60° angle, making it

$$\sin(60^\circ) = \frac{\sqrt{3}}{2}.$$

As such,

$$AB = 4 \cdot \frac{\sqrt{3}}{2} = 2\sqrt{3}.$$

Then, since it is an equilateral triangle, it has area

$$\frac{s^2\sqrt{3}}{4} = \frac{12\sqrt{3}}{4} = 3\sqrt{3}.$$

Thus, the correct answer is **B**.

14. Danica drove her new car on a trip for a whole number of hours, averaging 55 miles per hour. At the beginning of the trip, abc miles was displayed on the odometer, where abc is a 3-digit number with $a \geq 1$ and $a + b + c \leq 7$. At the end of the trip, the odometer showed cba miles. What is $a^2 + b^2 + c^2$?

A 26

B 27

C 36

D 37

E 41

Solution(s):

We know that the difference of the numbers cba and abc is equal to:

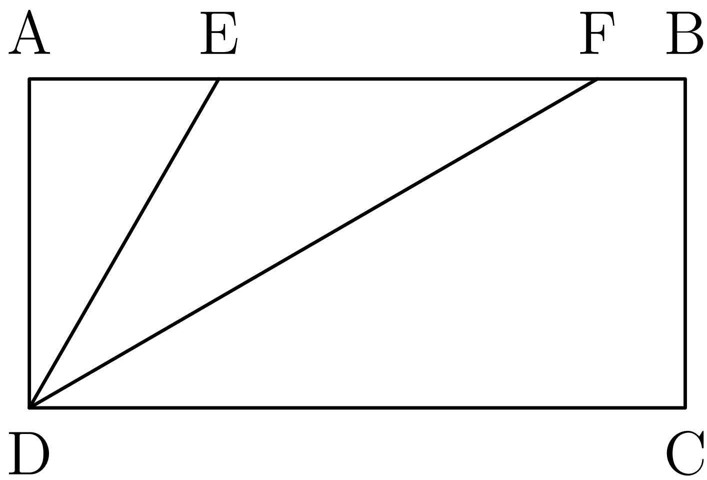
$$100c + 10b + a - 100a - 10b - c$$
$$= 99(c - a)$$

We know that this number also must be a multiple of 55. As $\gcd(55, 99)$ is 11, we know that $c - a$ is a multiple of 5, and $c > a$.

This makes $a = 1, b = 0, c = 6$ the only possible value with $a + b + c \leq 7$ as every other combination has $a + b + c > 7$. As such, $a^2 + b^2 + c^2 = 37$.

Thus, the correct answer is **D**.

15. In rectangle $ABCD$, $DC = 2 \cdot CB$ and points E and F lie on $\overline{AB}$ so that $\overline{ED}$ and $\overline{FD}$ trisect $\angle ADC$ as shown. What is the ratio of the area of $\triangle DEF$ to the area of rectangle $ABCD$?



A $\frac{\sqrt{3}}{6}$

B $\frac{\sqrt{6}}{8}$

C $\frac{3\sqrt{3}}{16}$

D $\frac{1}{3}$

E $\frac{\sqrt{2}}{4}$

Solution(s):

The area of EFB is equal to

$$\frac{EF \cdot AD}{2}.$$

Similarly, The area of $ABCD$ is equal to

$$AB \cdot AD.$$

Thus, their ratio is

$$\frac{EF}{2 \cdot AD} = \frac{EF}{4 \cdot AB}.$$

Then,

$$\begin{aligned} EF &= AF - AE \\ &= AB\tan(\angle ADE) \\ &\quad -AB\tan(\angle ADF) \\ &= AB(\tan(60^\circ) - \tan(30^\circ)) \\ &= AB\left(\sqrt{3} - \frac{\sqrt{3}}{3}\right) \\ &= AB\left(\frac{2\sqrt{3}}{3}\right) \end{aligned}$$

This makes our result

$$\frac{EF}{4 \cdot AB} = \frac{AB(\frac{2\sqrt{3}}{3})}{4 \cdot AB} = \frac{\sqrt{3}}{6}.$$

Thus, the correct answer is **A**.

16. Four fair six-sided dice are rolled. What is the probability that at least three of the four dice show the same value?

A $\frac{1}{36}$

B $\frac{7}{72}$

C $\frac{1}{9}$

D $\frac{5}{36}$

E $\frac{1}{6}$

Solution(s):

There are two cases: All of the numbers are the same or one of the numbers is different from the other ones.

Case 1 - All numbers are the same: The probability of this is

$$\frac{6}{6^4}$$

since there are 6 combinations where they are all the same due to the 6 possible values for the dice.

Case 2 - One of the numbers are: The probability of this is

$$\frac{4 \cdot 5 \cdot 6}{6^4}$$

since there are 4 ways to choose the dice that is different, 6 values for the dice that are the same, and 5 values for the other dice.

This makes the probability

$$\frac{6 \cdot 5 \cdot 4 + 6}{6^4} = \frac{21}{216} = \frac{7}{72}.$$

Thus, the correct answer is **B**.

17. What is the greatest power of 2 that is a factor of $10^{1002} - 4^{501}$?

A 2^{1002}

B 2^{1003}

C 2^{1004}

D 2^{1005}

E 2^{1006}

Solution(s):

Observe that:

$$\begin{aligned} 10^{1002} - 4^{501} &= 10^{1002} - 2^{1002} \\ &= 2^{1002}(5^{1002} - 1) \\ &= 2^{1002}(25^{501} - 1). \end{aligned}$$

Then, since

$$25^{501} \equiv 1^{501} \equiv 1 \mod 8,$$

We know that $25^{501} - 1$ is a multiple of 2^3, making our number a multiple of 2^{1005}.

However,

$$\begin{aligned} &25^{501} - 1 \\ &\equiv 9^{501} - 1 \mod 16 \\ &\equiv 9\left(81^{250}\right) - 1 \mod 16 \\ &\equiv 8 \mod 16, \end{aligned}$$

so $25^{501} - 1$ is not a multiple of 2^4, and as such, our number isn't a multiple of 2^{1006}.

Thus, the correct answer is **D**.

18. A list of 11 positive integers has a mean of 10, a median of 9, and a unique mode of 8. What is the largest possible value of an integer in the list?

A 24

B 30

C 31

D 33

E 35

Solution(s):

Since 9 is the median, there are 5 numbers less than or equal to 9 besides the middle.

Also, there are 5 numbers greater than or equal to 9 besides the middle.

Also, since 8 is the unique mode, it must show up at least twice.

Furthermore, since we are trying to maximize the largest number and we have their mean, we need to minimize the sum of the first 10 numbers. Finally, notice that the sum should be $11 \cdot 10 = 110$.

Now, we could case on the number of times 8 appears since every other number must show up less than it.

If it appears twice, every other number must show up at most once, so the maximum sum is

$$1+2+3+8+8+9$$
$$+10+11+12+13$$
$$=77.$$

This makes the largest number 33.

If it appears three times, every other number must show up at most twice, so the maximum sum is

$$1+1+8+8+8$$
$$+9+9+10+10+11$$
$$=75.$$

This makes the largest number 35.

If it appears four times, every other number must show up at most once, so the maximum sum is

$$1+8+8+8+8$$
$$+9+9+9+9+10$$
$$=79.$$

This makes the largest number 31.

This makes the maximum 35.

Thus, the correct answer is **E**.

19. Two concentric circles have radii 1 and 2. Two points on the outer circle are chosen independently and uniformly at random. What is the probability that the chord joining the two points intersects the inner circle?

A $\frac{1}{6}$

B $\frac{1}{4}$

C $\frac{2-\sqrt{2}}{2}$

D $\frac{1}{3}$

E $\frac{1}{2}$

Solution(s):

First, without loss of generality, we could choose some point on the outer circle. Then, the second point can be chosen in a region on the other circle.

This region is such that it has a line that intersects the circle, so the edge of the region is such that the chord is perpendicular with the inner circle.

If we look at the angle at the center, we can see that it has 2 right triangles where the adjacent side is 1 and the hypotenuse is 2, making

$$\cos\left(\frac{\theta}{2}\right) = \frac{1}{2}.$$

Thus, $\frac{\theta}{2} = 60^\circ$, making $\theta = 120^\circ$.

Therefore, the probability is

$$\frac{120^\circ}{360^\circ} = \frac{1}{3}.$$

Thus, the correct answer is **D**.

20. For how many integers x is the number $x^4 - 51x^2 + 50$ negative?

A 8

B 10

C 12

D 14

E 16

Solution(s):

First, note that

$$x^4 - 51x^2 + 50$$
$$= (x^2 - 50)(x^2 - 1).$$

If $(x^2 - 50)(x^2 - 1) < 0$ means that one of the terms is negative.

Since

$$x^2 - 50 < x^2 - 1,$$

it must be that

$$x^2 - 50 < 0, x^2 - 1 > 0.$$

This means $1 < x^2 < 50$, making

$$1 < |x| \leq 7,$$

resulting in 12 solutions.

Thus, the correct answer is **C**.

21. Trapezoid $ABCD$ has parallel sides $\overline{AB}$ of length 33 and $\overline{CD}$ of length 21. The other two sides are of lengths 10 and 14. The angles A and B are acute. What is the length of the shorter diagonal of $ABCD$?

A $10\sqrt{6}$

B 25

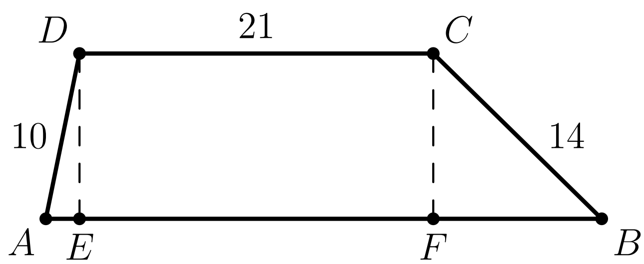
C $8\sqrt{10}$

D $18\sqrt{2}$

E 26

Solution(s):

Let the base of the altitude from C to AB be E. and let the base of the altitude from D to AB be F. Also, let $BC = 10$ since we can assign any value. This yields the following diagram:



Then, let $FB = x$ and the altitude be h. This means

$$\begin{aligned} AE &= 33 - x - EF \\ &= 33 - x - 21 \\ &= 12 - x. \end{aligned}$$

This suggests that:

$$14^2 = (12 - x)^2 + h^2$$

$$10^2 = x^2 + h^2.$$

Subtracting the equations, we get:

$$96 = 144 - 24x$$

$$x = 2.$$

Then, we want to find

$$\sqrt{(21+x)^2+h^2}$$

$$=\sqrt{21^2+42x+(x^2+h^2)}$$

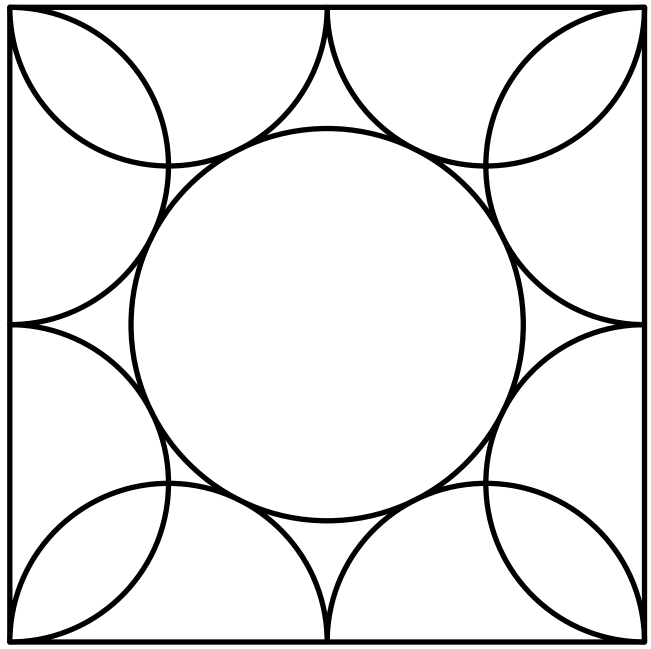
$$=\sqrt{441+42\cdot 2+100}$$

$$=\sqrt{625}$$

$$=25.$$

Thus, the correct answer is **B**.

22. Eight semicircles line the inside of a square with side length 2 as shown. What is the radius of the circle tangent to all of these semicircles?



A $\frac{1+\sqrt{2}}{4}$

B $\frac{\sqrt{5}-1}{2}$

C $\frac{\sqrt{3}+1}{4}$

D $\frac{2\sqrt{3}}{5}$

E $\frac{\sqrt{5}}{3}$

Solution(s):

The distance from the center of the square to the center of the semicircles can be found as a hypotenuse of a right triangle.

One of the legs is from the center of the square to the center of one of the sides which is of distance 1.

The other leg is from the center of the side to the center of one of the semicircles which is of distance $\frac{1}{2}$. This also shows that the radius of the semicircles is $\frac{1}{2}$. Therefore, the distance from the center of the square to the center of the semicircle is

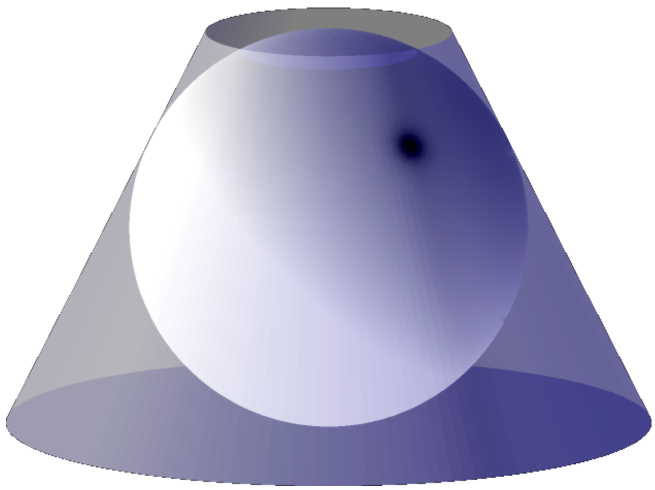
$$\sqrt{1^2 + \frac{1}{2}^2} = \frac{\sqrt{5}}{2}.$$

Then , we subtract $\frac{1}{2}$ for the radius of the semicircle. This makes the radius of the circle

$$\frac{\sqrt{5}-1}{2}.$$

Thus, the correct answer is **B**.

23. A sphere is inscribed in a truncated right circular cone as shown. The volume of the truncated cone is twice that of the sphere. What is the ratio of the radius of the bottom base of the truncated cone to the radius of the top base of the truncated cone?



A $\frac{3}{2}$

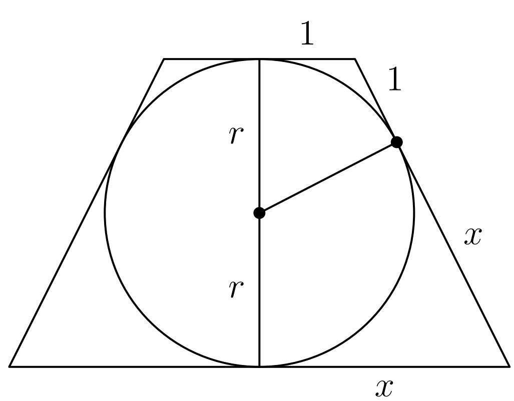
B $\frac{1+\sqrt{5}}{2}$

C $\sqrt{3}$

D 2

E $\frac{3+\sqrt{5}}{2}$

Solution(s):



Let the radius of the sphere be r. Then, let the top have a radius of 1 and let the bottom have a radius of x. Then, we could extend the cone to make a smaller cone on top with height h. This makes a larger cone with radius x and height $h+2r$.

Note that

$$\begin{aligned}\frac{h+2r}{h} &= x\\ \frac{2r}{h} &= x-1\\ h &= \frac{2r}{x-1}.\end{aligned}$$

This makes the volume of the whole cone

$$\frac{\pi(h+2r)(x^2)}{3},$$

and the volume of the top section

$$\frac{h}{3}\pi.$$

As such, the volume of the truncated cone is

$$\frac{hx^2+2rx^2-h}{3}\pi.$$

Then, by making a slice in the cone, we get the picture above. Then, by making two lines from the corners to the center, we get similar right triangles, which makes

$$\frac{r}{1}=\frac{x}{r}.$$

Thus, we get $r^2=x$.

This makes the volume

$$\frac{h(x^2-1)+2rx^2}{3}\pi$$

and

$$h = \frac{2r}{x-1} = \frac{2r(x+1)}{x^2-1}.$$

This further simplifies the volume to

$$\begin{aligned}&\frac{2r(x+1)+2rx^2}{3}\pi \\ &= \frac{2r(x^2+x+1)}{3}\pi.\end{aligned}$$

The volume of the sphere is $\frac{4r^3\pi}{3}$. This implies

$$\frac{8r^3\pi}{3} = \frac{2r(x^2+x+1)}{3}\pi.$$

Thus,

$$\begin{aligned}4x &= x^2+x+1 \\ x^2-3x+1 &= 0 \\ x &= \frac{3+\sqrt{5}}{2}.\end{aligned}$$

Since we made the top circle of radius 1, the ratio is

$$\frac{3+\sqrt{5}}{2}.$$

Thus, the correct answer is **E**.

24. The numbers $1, 2, 3, 4, 5$ are to be arranged in a circle. An arrangement is bad if it is not true that for every n from 1 to 15 one can find a subset of the numbers that appear consecutively on the circle that sum to n. Arrangements that differ only by a rotation or a reflection are considered the same. How many different bad arrangements are there?

- A 1
- B 2
- C 3
- D 4
- E $5.$

Solution(s):

We wil always have the numbers

$$1, 2, 3, 4, 5$$

by choosing the number, and we will always have

$$14, 13, 12, 11, 10$$

by choosing the entire set except one. Then, 15 is also chosen.

Therefore, we need to find

$$6, 7, 8, 9.$$

Furthermore, if we have 6 or 7, we can get 8 and 9 by choosing the rest.

This means a configuration is only not bad if and only if we can find a set of numbers put together that adds to 6 and 7.

The only ways to make a sum 6 is with a sequence of

$$(1, 2, 3), (1, 5), \text{ or } (2, 4).$$

The only ways to make a sum 7 is with a sequence of

$$(1, 2, 4), (2, 5), \text{ or } (3, 4).$$

Note that the triples can be in any order.

This means that for any configuration without one of them, we need to prevent all of the combinations for one of the numbers. Thus, we can case on which number isn't accounted for.

Case 1: There is no 6.

This means the 1 isn't next to the 5. Thus, it is next to the $2, 3$ or 4. However, it can't have both 2 and 3, so the 1 must be next to 4 and one of 2 and 3.

Also, this means the 2 isn't next to the 4. Thus, it is next to the $1, 3$ or 5. However, it can't have both 1 and 3, so the 2 must be next to 5 and one of 1 and 3.

Then, 5 must be next to the 2 and 3 since if it is connected to the 4, it would make a

$$1-2-3$$

which has a sum of 6. Thus, the configuration is starts with

$$3-5-2.$$

Since 2 isn't next to 4, we have

$$3-5-2-1-4$$

as the only configuration.

Case 2: There is no 7.

This means the 2 isn't next to the 5. Thus, it is next to the $1, 3$ or 4. However, it can't have both 1 and 4, so the 2 must be next to 4 and one of 1 and 3.

Also, this means the 4 isn't next to the 3. Thus, it is next to the $1, 2$ or 5. However, it can't have both 1 and 2, so the 4 must be next to 5 and one of 1 and 2.

Then, 5 must be next to the 1 and 4 since if it is connected to the 3, it would make a

$$1-2-4$$

which has a sum of 6. Thus, the configuration is starts with

$$1-5-4.$$

Since 4 isn't next to 3, we have

$$1-5-4-2-3$$

as the only configuration.

This means there are only two configurations.

Thus, the correct answer is **B**.

25. In a small pond there are eleven lily pads in a row labeled 0 through 10. A frog is sitting on pad 1. When the frog is on pad N, $0 < N < 10$, it will jump to pad $N-1$ with probability $\frac{N}{10}$ and to pad $N+1$ with probability $1-\frac{N}{10}$. Each jump is independent of the previous jumps.

If the frog reaches pad 0 it will be eaten by a patiently waiting snake. If the frog reaches pad 10 it will exit the pond, never to return. What is the probability that the frog will escape without being eaten by the snake?

A $\frac{32}{79}$

B $\frac{161}{384}$

C $\frac{63}{146}$

D $\frac{7}{16}$

E $\frac{1}{2}$

Solution(s):

Let the value e_i be the probability of winning at each time. Then, for all $1 \leq i \leq 9$, we have

$$e_i = \frac{10-i}{10}e_{i+1} + \frac{i}{10}e_{i-1}.$$

Also, by symmetry, we have $e_5 = \frac{1}{2}$. Our goal is to find e_1.

Then,

$$\begin{aligned} e_4 &= \frac{6}{10}e_5 + \frac{4}{10}e_3 \\ &= \frac{6}{10}(\frac{1}{2}) + \frac{4}{10}e_3 \\ &= \frac{3}{10} + \frac{4}{10}e_3. \end{aligned}$$

Then,

$$\begin{aligned} e_3 &= \frac{7}{10}e_4 + \frac{3}{10}e_2 \\ &= \frac{7}{10}(\frac{3}{10} + \frac{4}{10}e_3) + \frac{3}{10}e_2 \\ &= \frac{21}{100} + \frac{28}{100}e_3 + \frac{30}{100}e_2 \\ &= \frac{21}{72} + \frac{30}{72}e_2 \\ &= \frac{7}{24} + \frac{5}{12}e_2 \end{aligned}$$

Then,

$$\begin{aligned} e_2 &= \frac{8}{10}e_3 + \frac{2}{10}e_1 \\ &= \frac{8}{10}(\frac{7}{24} + \frac{5}{12}e_2) + \frac{2}{10}e_1 \\ &= \frac{56}{240} + \frac{40}{120}e_2 + \frac{2}{10}e_1 \\ &= \frac{7}{30} + \frac{1}{3}e_2 + \frac{1}{5}e_1 \\ &= e_2 = \frac{7}{20} + \frac{3}{10}e_1. \end{aligned}$$

Then, $e_1 = \frac{9}{10}e_2 + \frac{1}{10}e_0 = \frac{9}{10}(\frac{7}{20} + \frac{3}{10}e_1) = \frac{63}{200} + \frac{27}{100}e_1.$

This suggests that

$$\frac{73}{100}e_1 = \frac{63}{200}$$

$$e_1 = \frac{63}{146}.$$

Thus, the correct answer is **C**.

Problems: https://live.poshenloh.com/past-contests/amc10/2014B