2026 AIME II Solutions

Typeset by: LIVE by Po-Shen Loh
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1. Find the sum of the 10th terms of all arithmetic sequences of integers that have first term equal to 4 and include both 24 and 34 as terms.

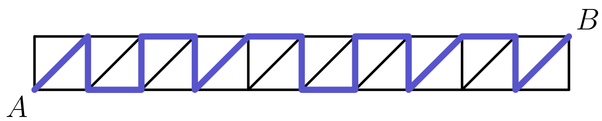
Solution:

Let the common difference be d. Since the first term is 4 and both 24 and 34 appear, d divides $24 - 4 = 20$ and $34 - 4 = 30$, so d divides $\gcd(20, 30) = 10$. The difference must be positive to reach 24 and 34 from 4, so $d \in \{1, 2, 5, 10\}$ (and each of these works, since $d \mid 20$ and $d \mid 30$ put both targets in the sequence).

The 10th term is $4 + 9d$, so the requested sum is

$$\sum_{d\in\{1,2,5,10\}} (4 + 9d) = 4 \cdot 4 + 9(1 + 2 + 5 + 10) = 16 + 162 = 178.$$

2. The figure below shows a grid of 10 squares in a row. Each square has a diagonal connecting its lower left vertex to its upper right vertex. A bug moves along the line segments from vertex to vertex, never traversing the same segment twice and never moving from right to left along a horizontal or diagonal segment. Let N be the number of paths the bug can take from the lower left corner (A) to the upper right corner (B). One such path from A to B is shown by the thick line segments in the figure. Find $\sqrt{N}$.



Solution:

Put $A = (0,0)$ and $B = (10,1)$. Every horizontal and diagonal move goes rightward, so the bug's x-coordinate never decreases, and it crosses each of the 10 vertical strips exactly once, using exactly one of that square's three rightward segments: the bottom edge, the top edge, or the diagonal.

These ten choices determine the whole path. Each crossing arrives at a definite height (bottom edge: low; top edge or diagonal: high) and departs at a definite height (bottom edge or diagonal: low; top edge: high), so at each vertical line the bug traverses the vertical segment exactly when the arrival and departure heights differ — and each vertical segment is needed at most once, so no segment repeats. The same applies at the ends: the bug starts low at A and finishes high at B, using the end verticals if necessary. Conversely, every sequence of choices yields a valid path.

Therefore $N = 3^{10}$, and $\sqrt{N} = 3^5 = 243.$

3. Let $ABCDE$ be a nonconvex pentagon with internal angles $\angle A = \angle E = 90^\circ$ and $\angle B = \angle D = 45^\circ$. Suppose that $DE < AB$, $AE = 20$, $BC = 14\sqrt{2}$, and points B, C, and D lie on the same side of line AE. Suppose further that AB is an integer with $AB < 2026$ and the area of pentagon $ABCDE$ is an integer multiple of 16. Find the number of possible values of AB.

Solution:

Place $A = (0,0)$ and $E = (20,0)$ with the pentagon above line AE, and write $h = AB$. The right angles at A and E make AB and ED vertical: $B = (0,h)$ and $D = (20,k)$ with $k = DE$. At B the side $BC = 14\sqrt{2}$ makes a 45° angle with the downward ray BA, heading into the pentagon, so $C = (14, h-14)$. Similarly at D, the side DC makes a 45° angle with the downward ray DE, so $C = (20-s, k-s)$ where $s = \frac{DC}{\sqrt{2}}$. Matching coordinates gives $s = 6$ and $k = h - 8$. The interior angle at C is then the reflex angle 270° (angle sum $90 + 45 + 270 + 45 + 90 = 540$), and $DE = h - 8 < AB$ automatically.

The shoelace formula on $A(0,0), B(0,h), C(14,h-14), D(20,h-8), E(20,0)$ gives area

$$\frac{1}{2}\left|-14h + (-6h + 168) + (-20h + 160)\right| = 20h - 164.$$

The condition $16 \mid 20h - 164$ reduces to $4h \equiv 4 \pmod{16}$, that is, $h \equiv 1 \pmod{4}$. For C to lie strictly on the same side of line AE as B and D, we need $h > 14$.

So h runs over $17, 21, 25, \ldots, 2025$, which is $\frac{2025-17}{4} + 1 = 503$ values.

4. For each positive integer n let $f(n)$ be the value of the base-ten numeral n viewed in base b, where b is the least integer greater than the greatest digit in n. For example, if $n = 72$, then $b = 8$, and 72 as a numeral in base 8 equals $7 \cdot 8 + 2 = 58$; therefore $f(72) = 58$. Find the number of positive integers n less than 1000 such that $f(n) = n$.

Solution:

If n has a single digit d, then the numeral d has value d in every base, so $f(n) = n$: all 9 one-digit numbers work. If n has digits $d_{k-1} \dots d_1 d_0$ with $k \geq 2$, then $b \leq 10$ always, and if $b < 10$ then $f(n) = \sum d_i b^i < \sum d_i 10^i = n$ because the leading digit satisfies $d_{k-1} b^{k-1} < d_{k-1} 10^{k-1}$. So a multi-digit n satisfies $f(n) = n$ exactly when $b = 10$, that is, when some digit of n equals 9.

Two-digit numbers containing a 9 : the numbers 90 through 99 plus $19, 29, \dots, 89$, for $10 + 8 = 18$. Three-digit numbers containing a 9 : $900 - 8 \cdot 9 \cdot 9 = 252$, subtracting the numbers with no 9 (leading digit 1–8, others 0–8).

The total is $9 + 18 + 252 = 279$.

5. An urn contains n marbles. Each marble is either red or blue, and there are at least 7 marbles of each color. When 7 marbles are drawn randomly from the urn without replacement, the probability that exactly 4 of them are red equals the probability that exactly 5 of them are red. Find the sum of the five least values of n for which this is possible.

Solution:

Say there are r red and b blue marbles, $r, b \geq 7$. The condition is $\binom{r}{4}\binom{b}{3} = \binom{r}{5}\binom{b}{2}$. Since $\binom{r}{5} = \binom{r}{4}\frac{r-4}{5}$ and $\binom{b}{3} = \binom{b}{2}\frac{b-2}{3}$, cancelling gives

$$\frac{b-2}{3} = \frac{r-4}{5}, \qquad \text{that is,} \qquad b = \frac{3r-2}{5}.$$

So $r \equiv 4 \pmod{5}$, and $b \geq 7$ requires $3r - 2 \geq 35$, so $r \geq 14$. The five smallest choices are $r = 14, 19, 24, 29, 34$ with $b = 8, 11, 14, 17, 20$, giving $n = 22, 30, 38, 46, 54$.

The sum is $22 + 30 + 38 + 46 + 54 = 190$.

6. Find the sum of all real numbers r such that there is at least one point where the circle with radius r centered at $(4, 39)$ is tangent to the parabola with equation $2y = x^2 - 8x + 12$.

Solution:

Completing the square, $2y = (x-4)^2 - 4$, so with $u = x - 4$ the parabola is the set of points $\left(4 + u,\ \frac{u^2}{2} - 2\right)$ and the center $(4, 39)$ lies on its axis. The circle is tangent to the parabola at a point exactly when the two curves share a tangent line there, i.e. when the radius to that point is normal to the parabola — which happens exactly at critical points of the squared distance

$$D(u) = u^2 + \left(\frac{u^2}{2} - 41\right)^2, \qquad D'(u) = 2u + u\left(u^2 - 82\right) = u\left(u^2 - 80\right).$$

At $u = \pm\sqrt{80} : D = 80 + (40 - 41)^2 = 81$, so $r = 9$ (the circle touches the parabola at two symmetric points). At $u = 0$, the point is the vertex $(4, -2)$ at distance 41, where the parabola and the circle of radius 41 both have horizontal tangent lines, so $r = 41$ also works.

The sum is $9 + 41 = 50$.

7. A standard fair six-sided die is rolled repeatedly. Each time the die reads 1 or 2, Alice gets a coin; each time it reads 3 or 4, Bob gets a coin; and each time it reads 5 or 6, Carol gets a coin. The probability that Alice and Bob each receive at least two coins before Carol receives any coins can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $100m + n$.

Solution:

Each roll is an Alice roll, a Bob roll, or a Carol roll, each with probability $\frac{1}{3}$. The event succeeds exactly when the rolls before the first Carol roll include at least two Alice rolls and at least two Bob rolls. The first Carol roll is roll $k+1$ with probability $\left(\frac{2}{3}\right)^k \frac{1}{3}$, and given this, the first k rolls form one of 2^k equally likely Alice/Bob strings. For $k \geq 3$ the bad strings — at most one Alice, or at most one Bob — number $(k+1) + (k+1) = 2k+2$, and no string is bad in both ways. Hence

$$P = \sum_{k\geq 4} \frac{1}{3}\left(\frac{2}{3}\right)^k \left(1 - \frac{2k+2}{2^k}\right).$$

The first piece is $\sum_{k\geq 4}\left(\frac{2}{3}\right)^k = \frac{16}{27}$. For the second, $\sum_{k\geq 0}\frac{k+1}{3^k} = \frac{1}{(1-1/3)^2} = \frac{9}{4}$, so

$$\sum_{k\geq 4}\frac{2k+2}{3^k} = 2\left(\frac{9}{4} - 1 - \frac{2}{3} - \frac{1}{3} - \frac{4}{27}\right) = 2\cdot\frac{11}{108} = \frac{11}{54}.$$

Therefore $P = \frac{1}{3}\left(\frac{16}{27} - \frac{11}{54}\right) = \frac{1}{3}\cdot\frac{21}{54} = \frac{7}{54}$, and $100m + n = 700 + 54 = 754.$

8. Isosceles triangle $\triangle ABC$ has $AB = BC$. Let I be the incenter of $\triangle ABC$. The perimeters of $\triangle ABC$ and $\triangle AIC$ are in the ratio $125 : 6$, and all the sides of both triangles have integer lengths. Find the minimum possible value of AB.

Solution:

Let $a = AB = BC$ and $b = AC$, so $s = a + \frac{b}{2}$. The incircle touches AC at its midpoint (tangent length from A is $s - a = \frac{b}{2}$), so $AI^2 = CI^2 = r^2 + \frac{b^2}{4}$. By Heron's formula, $r^2 = \frac{(s-a)^2(s-b)}{s} = \frac{b^2}{4} \cdot \frac{2a-b}{2a+b}$, and therefore

$$AI^2 = \frac{b^2}{4}\left(\frac{2a-b}{2a+b} + 1\right) = \frac{ab^2}{2a+b}.$$

The perimeter condition is $125\,(2\,AI + b) = 6\,(2a + b)$.

Since AI is rational, write $\sqrt{\frac{a}{2a+b}} = \frac{p}{q}$ in lowest terms. Then $aq^2 = p^2(2a+b)$ forces $p^2 \mid a$; writing $a = mp^2$ gives $b = m(q^2 - 2p^2)$, $2a + b = mq^2$, and $AI = \frac{mp(q^2-2p^2)}{q}$. The perimeter condition then loses m entirely:

$$125\,(q^2 - 2p^2)(2p + q) = 6q^3.$$

Since $\gcd(125, 6) = 1$ we get $5 \mid q$; and q must be even, since for odd q both factors on the left are odd while the right side is even. Writing $q = 10w$ and simplifying, $(50w^2 - p^2)(p + 5w) = 12w^3$. Both factors on the left are coprime to w (as $\gcd(p, q) = 1$), so $w = 1$, and $(50 - p^2)(p + 5) = 12$ has the unique solution $p = 7$.

So $a = 49m, b = 2m$, and $AI = \frac{7m}{5}$, which is an integer exactly when $5 \mid m$. Taking $m = 5$ gives $\triangle ABC$ with sides $245, 245, 10$ and $\triangle AIC$ with sides $7, 7, 10$, whose perimeters 500 and 24 are indeed in ratio $125 : 6$. The minimum possible AB is 245.

9. Let S denote the value of the infinite sum

$$\frac{1}{9}+\frac{1}{99}+\frac{1}{999}+\frac{1}{9999}+\cdots$$

Find the remainder when the greatest integer less than or equal to $10^{100}S$ is divided by $1000.$

Solution:

Each term is $\frac{1}{10^k-1}=\sum_{j\ge1}10^{-kj}$, so summing over k and collecting the exponent $n=kj$,

$$S=\sum_{n\ge1}\frac{d(n)}{10^n},$$

where $d(n)$ is the number of divisors of n. Hence $10^{100}S=\sum_{n=1}^{100}d(n)\,10^{100-n}+T$ with $T=\sum_{m\ge1}d(100+m)\,10^{-m}$.

From $d(101)=2, d(102)=8, d(103)=2, d(104)=8$, the tail starts $0.2+0.08+0.002+0.0008=0.2828$, and since $d(N)<2\sqrt{N}$, the remaining terms contribute less than $\sum_{m\ge5}\frac{2\sqrt{100+m}}{10^m}<0.001$. So $0<T<1$ and

$$\lfloor 10^{100}S\rfloor=\sum_{n=1}^{100}d(n)\,10^{100-n}.$$

Modulo 1000, every term with $n\le97$ is a multiple of 1000, leaving $d(98)\cdot100+d(99)\cdot10+d(100)$. Since $d(98)=6, d(99)=6$, and $d(100)=9$, the remainder is $600+60+9=669$.

10. Let $\triangle ABC$ be a triangle with D on $\overline{BC}$ such that $\overline{AD}$ bisects $\angle BAC$. Let ω be the circle that passes through A and is tangent to segment $\overline{BC}$ at D. Let $E \neq A$ and $F \neq A$ be the intersections of ω with segments $\overline{AB}$ and $\overline{AC}$, respectively. Suppose that $AB = 200$, $AC = 225$, and all of AE, AF, BD, and CD are positive integers. Find the greatest possible value of BC.

Solution:

Since ω is tangent to BC at D, the power of B gives $BD^2 = BE \cdot BA$ and the power of C gives $CD^2 = CF \cdot CA$. The angle bisector gives $\frac{BD}{DC} = \frac{AB}{AC} = \frac{8}{9}$, so $BD = 8t$ and $CD = 9t$, where $t = CD - BD$ is a positive integer. Then

$$BE = \frac{64t^2}{200} = \frac{8t^2}{25}, \qquad CF = \frac{81t^2}{225} = \frac{9t^2}{25},$$

so $AE = 200 - \frac{8t^2}{25}$ and $AF = 225 - \frac{9t^2}{25}$.

For AE and AF to be integers we need $25 \mid t^2$, that is, $t = 5s$. Then $AE = 200 - 8s^2 > 0$ forces $s \leq 4$, and $BC = 17t = 85s$. At $s = 4 : BC = 340$, with $BD = 160, CD = 180$, $AE = 72, AF = 81$ all positive integers, and the sides $200, 225, 340$ form a valid triangle since $200 + 225 > 340$.

The greatest possible value of BC is 340.

11. Find the greatest integer n such that the cubic polynomial

$$x^3 - \frac{n}{6}x^2 + (n-11)x - 400$$

has roots α^2, β^2, and γ^2, where α, β, and γ are complex numbers, and there are exactly seven different possible values for $\alpha + \beta + \gamma$.

Solution:

The roots of the cubic are $\alpha^2, \beta^2, \gamma^2$. Fix square roots s_1, s_2, s_3 of them; then $\alpha + \beta + \gamma$ ranges over the eight expressions $\pm s_1 \pm s_2 \pm s_3$, which come in four pairs $\pm v$. Generically all eight are distinct. A coincidence $v(\varepsilon) = v(\varepsilon')$ between choices that are not opposite forces $s_i = \pm s_j$ for some $i \neq j$, which collapses the eight values to at most six. So exactly seven values occur precisely when one choice satisfies $\pm s_1 \pm s_2 \pm s_3 = 0$ – its opposite is then the same value 0 – and no further degeneracies occur.

That condition is the vanishing of

$$(s_1+s_2+s_3)(-s_1+s_2+s_3)(s_1-s_2+s_3)(s_1+s_2-s_3) = 2\sum_{i<j} r_i r_j - \sum_i r_i^2 = 4e_2 - e_1^2,$$

where $r_i = s_i^2$ are the roots and e_1, e_2 their elementary symmetric functions. By Vieta's formulas $e_1 = \frac{n}{6}$ and $e_2 = n - 11$, so $\frac{n^2}{36} = 4(n-11)$, i.e. $n^2 - 144n + 1584 = 0$, with roots $n = 12$ and $n = 132$.

For $n = 132$ the cubic's roots are distinct and nonzero (the constant term is $400 \neq 0$), so the only coincidence is the value 0 and exactly seven sums occur. The greatest such integer is 132.

12. Consider a tetrahedron with two isosceles triangle faces with side lengths $5\sqrt{10}, 5\sqrt{10}$, and 10 and two isosceles triangle faces with side lengths $5\sqrt{10}, 5\sqrt{10}$, and 18. The four vertices of the tetrahedron lie on a sphere with center S, and the four faces of the tetrahedron are tangent to a sphere with center R. The distance RS can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m+n$.

Solution:

The four faces have side multiset $\{5\sqrt{10} \times 8,\ 10 \times 2,\ 18 \times 2\}$, and each edge lies on two faces, so the tetrahedron $ABCD$ has $AB = 10$ and $CD = 18$ as opposite edges and the other four edges equal to $5\sqrt{10}$. Place

$$A = (-5, 0, 12), \quad B = (5, 0, 12), \quad C = (0, -9, 0), \quad D = (0, 9, 0),$$

which is consistent since $AC^2 = 25 + 81 + 144 = 250 = (5\sqrt{10})^2$. The configuration is symmetric under $x \to -x$ and under $y \to -y$, so both centers lie on the z-axis.

For $S = (0, 0, s)$, equating distances to A and C gives $25 + (12 - s)^2 = 81 + s^2$, so $s = \frac{11}{3}$. For $R = (0, 0, t)$, face ABC has plane $4y - 3z + 36 = 0$ and face ACD has plane $12x + 5z = 0$, so equal distances require

$$\frac{36 - 3t}{5} = \frac{5t}{13} \implies t = \frac{117}{16},$$

and by the two mirror symmetries this point is equidistant (at distance $\frac{45}{16}$) from all four faces.

Therefore $RS = \frac{117}{16} - \frac{11}{3} = \frac{351 - 176}{48} = \frac{175}{48}$, which is in lowest terms, so $m + n = 175 + 48 = 223$.

13. Call finite sets of integers S and T *cousins* if

- S and T have the same number of elements,
- S and T are disjoint, and
- the elements of S can be paired with the elements of T so that the elements in each pair differ by exactly 1.

For example, $\{1, 2, 5\}$ and $\{0, 3, 4\}$ are cousins. Suppose that the set S has exactly 4040 cousins. Find the least number of elements the set S can have.

Solution:

A cousin T is the image of an injection sending each $x \in S$ to $x - 1$ or $x + 1$, landing outside S. If $x - 1, x, x + 1 \in S$ then x has nowhere to go, so every maximal block of consecutive elements of S has size 1 or 2. A double block $\{a, a + 1\}$ is forced to map to $\{a - 1, a + 2\}$, while a singleton $\{a\}$ chooses $a - 1$ or $a + 1$. Two blocks can fight over a value only when exactly one integer separates them, so group blocks into chains: consecutive blocks with gaps of exactly one. Within a chain the only consistent patterns are "the first i blocks shift left and the rest shift right," since a block choosing right and its successor choosing left would collide; a double block acts as both left and right, forcing the switch to happen exactly at it. Hence a chain of k singletons produces $k + 1$ distinct images, a chain containing one double produces exactly 1, and a chain with two doubles produces 0. Distinct patterns give distinct sets T, and choices in different chains are independent, so the number of cousins is the product of $(k_i + 1)$ over the all-singleton chains.

We need $\prod(k_i + 1) = 4040 = 2^3 \cdot 5 \cdot 101$ while minimizing the element count $\sum k_i$ (chains with doubles only waste elements). Replacing a composite factor $f = gh$ with the two factors $g, h \geq 2$ strictly lowers the cost, because $(g - 1) + (h - 1) < gh - 1$. So the optimum uses the prime factorization:

$$\sum(f - 1) = 1 + 1 + 1 + 4 + 100 = 107,$$

realized by five chains of $1, 1, 1, 4, 100$ singletons — runs of every-other integer — placed far apart.

The least possible number of elements is 107.

14. For integers a and b, let $a \circ b = a - b$ if a is odd and b is even, and $a \circ b = a + b$ otherwise. Find the number of sequences $a_1, a_2, a_3, \ldots, a_n$ of positive integers such that

$$a_1 + a_2 + a_3 + \cdots + a_n = 12 \quad \text{and} \quad a_1 \circ a_2 \circ a_3 \circ \cdots \circ a_n = 0,$$

where the operations are performed from left to right; that is, $a_1 \circ a_2 \circ a_3$ means $(a_1 \circ a_2) \circ a_3$.

Solution:

Since $a - b \equiv a + b \pmod 2$, the running value after k steps has the same parity as $a_1 + \cdots + a_k$. So term a_k is subtracted exactly when a_k is even and the prefix sum $a_1 + \cdots + a_{k-1}$ is odd, and the final value is 12 minus twice the total of the subtracted terms. We must count compositions of 12 in which the even terms sitting where the prefix sum is odd total exactly 6. The prefix parity flips exactly at odd terms, so the odd terms come in $2m$ (the total is even), and the subtracted terms are precisely the even terms lying between the $(2i-1)$st and $2i$th odd terms; these m "odd stretches" must hold even terms totaling 6, while the other $m+1$ stretches hold even terms totaling $6 - A$, where A is the sum of the odd terms.

Let $f_r(t)$ be the number of ways to fill r ordered stretches with sequences of even terms totaling $2t$. One stretch is a composition of $2t$ into even parts, i.e. of $t: f_1(t) = 2^{t-1}$ for $t \geq 1$ and $f_1(0) = 1$; convolving gives the values needed below: $f_r(0) = 1, f_r(1) = r, f_2(2) = 5$, and $f_1(3), f_2(3), f_3(3) = 4, 12, 25$. Compositions of A into $2m$ odd parts number $\binom{(A-2m)/2+2m-1}{2m-1}$.

Casework on m and $A:$ for $m = 1: A = 2, 4, 6$ give $1 \cdot 4 \cdot f_2(2) = 20, 2 \cdot 4 \cdot f_2(1) = 16$, and $3 \cdot 4 \cdot 1 = 12$. For $m = 2: A = 4, 6$ give $1 \cdot 12 \cdot f_3(1) = 36$ and $4 \cdot 12 \cdot 1 = 48$. For $m = 3:$ $A = 6$ gives $1 \cdot 25 \cdot 1 = 25$. The total is $20 + 16 + 12 + 36 + 48 + 25 = 157$.

15. Find the number of ordered 7-tuples $(a_1, a_2, a_3, \ldots, a_7)$ having the following properties:

- $a_k \in \{1, 2, 3\}$ for all k.
- $a_1 + a_2 + a_3 + a_4 + a_5 + a_6 + a_7$ is a multiple of 3.
- $a_1a_2a_4 + a_2a_3a_5 + a_3a_4a_6 + a_4a_5a_7 + a_5a_6a_1 + a_6a_7a_2 + a_7a_1a_3$ is a multiple of 3.

Solution:

Work modulo 3 : entries 3 are 0 and entries $1, 2$ are ± 1. Because the differences of $\{0, 1, 3\}$ hit every nonzero residue mod 7 exactly once, the seven triples $\{i, i+1, i+3\}$ are the lines of a Fano plane on the positions: every pair of positions lies on exactly one line, and any two lines meet in exactly one point. Let Z be the set of positions holding a 3 and $k = |Z|$. A product term survives exactly when its line avoids Z, contributing $(-1)^{\#\{2\text{'s on the line}\}}$, and the linear condition constrains the $7-k$ values ± 1 to sum to 0 mod 3.

Casework on k. $k = 7$: the all-3s tuple works: 1. $k = 6$: a single ± 1 can't sum to 0 : none. $k = 5$: no line survives; the two nonzero entries must be a 1 and a 2 : $\binom{7}{2} \cdot 2 = 42$. $k = 4$: three ± 1s sum to 0 only if all equal, and the three nonzero positions must not form a line, else its product is ± 1 : $(35 - 7) \cdot 2 = 56$. $k = 3$: four ± 1s must split two and two; exactly one line avoids a non-line Z (spoiling the sum), while a line Z is avoided by no line: $7 \cdot \binom{4}{2} = 42$. $k = 2$: five ± 1s must go four and one; exactly two lines avoid Z, meeting at a point p and covering the five positions, and their products cancel exactly when the lone minority value avoids p : $\binom{7}{2} \cdot 2 \cdot 4 = 168$. $k = 1$: six ± 1s sum to 0 if all equal or three of each; the four lines avoiding Z pairwise meet in the six nonzero positions, and since the product of all four line-products is $+1$, we need exactly two negative lines. All-equal gives 0 or 4 negative lines; for three 2's, viewing positions as edges of K_4 on the four lines, a line is negative exactly when it has odd degree in the chosen 3-edge set, and exactly the 12 three-edge paths (of the $\binom{6}{3} = 20$ subsets) give two odd degrees: $7 \cdot 12 = 84$. $k = 0$: seven ± 1s need two or five 2's, which make 4 or 3 lines negative respectively, but $7 - 2t \equiv 0 \pmod 3$ needs $t \equiv 2 \pmod 3$: none.

The total is $1 + 42 + 56 + 42 + 168 + 84 = 393$.

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