2026 AIME I Solutions

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1. Patrick started walking at a constant speed along a straight road from his school to the park. One hour after Patrick left, Tanya started running at a constant speed of 2 miles per hour faster than Patrick walked, following the same straight road from the school to the park. One hour after Tanya left, José started bicycling at a constant speed of 7 miles per hour faster than Tanya ran, following the same straight road from the school to the park. All three people arrived at the park at the same time. The distance from the school to the park is $\frac{m}{n}$ miles, where m and n are relatively prime positive integers. Find $m+n$.

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Solution:

Let v be Patrick's speed in miles per hour and T his travel time in hours. Then Tanya travels for $T-1$ hours at speed $v+2$, and José travels for $T-2$ hours at speed $v+9$ (which is 7 more than Tanya's speed). Since all three cover the same distance,

$$vT = (v+2)(T-1) = (v+9)(T-2).$$

Expanding the first equality gives $0 = 2T - v - 2$, so $v = 2T - 2$. Expanding the second gives $0 = 9T - 2v - 18$, so $2v = 9T - 18$. Substituting, $4T - 4 = 9T - 18$, hence $T = \frac{14}{5}$ and $v = \frac{18}{5}$.

The distance is $vT = \frac{18}{5} \cdot \frac{14}{5} = \frac{252}{25}$, which is in lowest terms, so $m + n = 252 + 25 = 277$.

2. Find the number of positive integer palindromes written in base 10, with no zero digits, and whose digits add up to 13. For example, 42124 has these properties. Recall that a palindrome is a number whose representation reads the same from left to right as from right to left.

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Solution:

A palindrome with an even number of digits has each digit appearing in a mirrored pair, so its digit sum is even. Since 13 is odd, the palindrome has an odd number of digits, and if m is the middle digit, the rest of the digit sum $13 - m$ is split evenly between the two halves, so m is odd. A one-digit palindrome would need $m = 13$, which is impossible.

The palindrome is determined by its middle digit m and the block of digits to the left of center: a nonempty string of nonzero digits with sum $s = \frac{13-m}{2}$. For $m = 1, 3, 5, 7, 9$ we get $s = 6, 5, 4, 3, 2$. Since $s \leq 6$, every digit of such a string is automatically at most 9, so the number of strings is the number of compositions of s, which is 2^{s-1} (each of the $s - 1$ gaps between units is either a break or not).

The total is $2^5 + 2^4 + 2^3 + 2^2 + 2^1 = 32 + 16 + 8 + 4 + 2 = 62.$

3. A hemisphere with radius 200 sits on top of a horizontal circular disk with radius 200, and the hemisphere and disk have the same center. Let $\mathcal{T}$ be the region of points P in the disk such that a sphere of radius 42 can be placed on top of the disk at P and lie completely inside the hemisphere. The area of $\mathcal{T}$ divided by the area of the disk is $\frac{p}{q}$, where p and q are relatively prime positive integers. Find $p + q$.

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Solution:

A sphere of radius 42 resting on the disk at P has its center 42 directly above P. It lies inside the hemisphere of radius 200 exactly when its center is within $200 - 42 = 158$ of the common center O. If d is the distance from O to P, the center of the sphere is at distance $\sqrt{d^2 + 42^2}$ from O, so the condition is $d^2 + 42^2 \le 158^2$.

By difference of squares, $d^2 \le 158^2 - 42^2 = 116 \cdot 200 = 23200$. Thus $\mathcal{T}$ is a disk of radius $\sqrt{23200}$, and the ratio of areas is

$$\frac{23200}{200^2} = \frac{23200}{40000} = \frac{29}{50}.$$

Therefore $p + q = 29 + 50 = 79.$

4. Find the number of integers less than or equal to 100 that are equal to $a+b+ab$ for some choice of distinct positive integers a and b.

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Solution:

Since $a+b+ab=(a+1)(b+1)-1$, an integer n is representable exactly when $n+1=xy$ for distinct integers $x=a+1$ and $y=b+1$ that are each at least 2. So we count integers $n+1$ in $\{2,3,\ldots,101\}$ that admit such a factorization.

A prime has no factorization into two factors that are both at least 2, and the square of a prime p^2 factors that way only as $p \cdot p$, which is not allowed. Every other composite M works: if p is its smallest prime factor, then $M = p \cdot \frac{M}{p}$ with $\frac{M}{p} > p$ since $M > p^2$. In $\{2,\ldots,101\}$ there are 26 primes (the 25 primes below 100, together with 101) and 4 prime squares $(4,9,25,49)$.

The count is $100-26-4=70$.

5. A plane contains points A and B with $AB = 1$. Point A is rotated in the plane counterclockwise through an acute angle θ around point B to point A'. Then B is rotated in the plane clockwise through angle θ around point A' to point B'. Suppose $AB' = \frac{4}{3}$. The value of $\cos\theta$ can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

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Solution:

Work in the complex plane with $B = 0$ and $A = 1$. Rotating z about P through angle φ counterclockwise gives $P + e^{i\varphi}(z - P)$. So $A' = e^{i\theta}$, and rotating B clockwise through θ about A' gives

$$B' = A' + e^{-i\theta}(0 - A') = e^{i\theta} - e^{-i\theta}e^{i\theta} = e^{i\theta} - 1.$$

Then

$$AB'^2 = \left|e^{i\theta} - 2\right|^2 = (\cos\theta - 2)^2 + \sin^2\theta = 5 - 4\cos\theta.$$

Setting this equal to $\left(\frac{4}{3}\right)^2 = \frac{16}{9}$ gives $4\cos\theta = 5 - \frac{16}{9} = \frac{29}{9}$, so $\cos\theta = \frac{29}{36}$ (indeed positive, consistent with θ acute). Thus $m + n = 29 + 36 = 65$.

6. The product of all positive real numbers x satisfying the equation

$$\sqrt[20]{x^{\log_{2026} x}} = 26x$$

is an integer P. Find the number of positive integer divisors of P.

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Solution:

Let $t = \log_{2026} x$. Taking $\log_{2026}$ of both sides of $x^{(\log_{2026} x)/20} = 26x$ gives

$$\frac{t^2}{20} = \log_{2026} 26 + t, \qquad \text{that is} \qquad t^2 - 20t - 20\log_{2026} 26 = 0.$$

The discriminant $400 + 80\log_{2026} 26$ is positive, so there are two real roots t_1, t_2, each giving a valid positive solution $x = 2026^t$.

By Vieta's formulas $t_1 + t_2 = 20$, so the product of the solutions is $2026^{t_1} \cdot 2026^{t_2} = 2026^{20}$. Since $2026 = 2 \cdot 1013$ and 1013 is prime, $P = 2^{20} \cdot 1013^{20}$ has $21 \cdot 21 = 441$ positive divisors.

7. Find the number of functions π mapping the set $A = \{1, 2, 3, 4, 5, 6\}$ onto A such that for every $a \in A$,

$$\pi(\pi(\pi(\pi(\pi(\pi(a)))))) = a.$$

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Solution:

A function from a finite set onto itself is a bijection, so π is a permutation of six elements, and the condition says π^6 is the identity. A permutation satisfies $\pi^6 = \mathrm{id}$ exactly when every cycle in its cycle decomposition has length dividing 6. Among the possible lengths 1 through 6, only 4 and 5 fail to divide 6.

We subtract the permutations containing a 4-cycle or a 5-cycle from $6! = 720$. Cycle type $4 + 1 + 1$ gives $\frac{6!}{4 \cdot 2!} = 90$, type $4 + 2$ gives $\frac{6!}{4 \cdot 2} = 90$, and type $5 + 1$ gives $\frac{6!}{5} = 144$, for $90 + 90 + 144 = 324$ excluded permutations.

The count is $720 - 324 = 396$.

8. Let N be the number of positive integer divisors of 17017^{17} that leave a remainder of 5 upon division by 12. Find the remainder when N is divided by 1000.

Solution:

Since $17017 = 7 \cdot 11 \cdot 13 \cdot 17$, the divisors of 17017^{17} are $7^a 11^b 13^c 17^d$ with each exponent between 0 and 17. Modulo 12 we have $13 \equiv 1$, and $7^2 \equiv 11^2 \equiv 17^2 \equiv 1$ (as $17 \equiv 5$), so the residue of a divisor is $7^\alpha\, 11^\beta\, 5^\delta \pmod{12}$, where α, β, δ are the parities of a, b, d.

The four possible values $1, 5, 7, 11$ multiply like the group $\{1, 5, 7, 11\}$ mod 12, in which $7 \cdot 11 \equiv 5$. Checking the eight parity patterns, the residue is 5 exactly when $(\alpha, \beta, \delta) = (0, 0, 1)$ or $(1, 1, 0)$. Each parity condition is satisfied by 9 of the 18 choices of that exponent, while c is free with 18 choices.

Therefore $N = 2 \cdot 9 \cdot 9 \cdot 9 \cdot 18 = 26244$, and the remainder mod 1000 is 244.

9. Joanne has a blank fair six-sided die and six stickers each displaying a different integer from 1 to 6. Joanne rolls the die and then places the sticker labeled 1 on the top face of the die. She then rolls the die again, places the sticker labeled 2 on the top face, and continues this process to place the rest of the stickers in order. If the die ever lands with a sticker already on its top face, the new sticker is placed to cover the old sticker. Let p be the conditional probability that at the end of the process exactly one face has been left blank, given that all the even-numbered stickers are visible on faces of the die. Then p can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

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Solution:

Let $f_1, \ldots, f_6$ be the top faces rolled, independent and uniform over the six faces. Sticker i goes on face f_i and ends up visible exactly when $f_j \neq f_i$ for all $j > i$ (sticker 6 is always visible). So the conditioning event is $f_3, f_4, f_5, f_6 \neq f_2$ and $f_5, f_6 \neq f_4$. Counting choices in the order $f_1, f_2, f_3, f_4, f_5, f_6$ gives $6 \cdot 6 \cdot 5 \cdot 5 \cdot 4 \cdot 4 = 14400$ sequences out of 6^6.

A face is blank exactly when it never appears among $f_1, \ldots, f_6$, so exactly one blank face means the sequence takes exactly 5 distinct values, i.e. there is exactly one coincidence $f_i = f_j$ with $i < j$ and all other values distinct. The coincidence must not violate the conditioning: pairs $(2, j)$ and $(4, 5), (4, 6)$ are forbidden, leaving the 9 pairs $(1, 2), (1, 3), (1, 4), (1, 5), (1, 6), (3, 4), (3, 5), (3, 6), (5, 6)$. For each allowed pair, the five distinct values can be assigned in $6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 = 720$ ways, and every constraint holds automatically because the only repeated value occupies an allowed pair. That gives $9 \cdot 720 = 6480$ sequences.

Therefore $p = \frac{6480}{14400} = \frac{9}{20}$, and $m + n = 9 + 20 = 29$.

10. Let $\triangle ABC$ have side lengths $AB = 13, BC = 14$, and $CA = 15$. Triangle $\triangle A'B'C'$ is obtained by rotating $\triangle ABC$ about its circumcenter so that $\overline{A'C'}$ is perpendicular to $\overline{BC}$, with A' and B not on the same side of line $B'C'$. Find the integer closest to the area of hexagon $AA'CC'BB'$.

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Solution:

Place $B = (0,0), C = (14,0), A = (5,12)$. The circumcenter lies on $x = 7$, and equating distances to B and A gives $O = \left(7, \frac{33}{8}\right)$. The direction of $\overline{AC}$ is $C - A = (9, -12)$, parallel to $(3, -4)$. A rotation through φ makes $\overline{A'C'}$ vertical exactly when it sends $(3, -4)$ to $(0, \pm 5)$, so $(\cos\varphi, \sin\varphi) = \left(\frac{4}{5}, -\frac{3}{5}\right)$ or $\left(-\frac{4}{5}, \frac{3}{5}\right)$. Rotating each vertex about O and checking the line $B'C'$ shows that A' and B are on opposite sides only for $\cos\varphi = \frac{4}{5}, \sin\varphi = -\frac{3}{5}$.

With this rotation, $P' = O + R(P - O)$ gives

$$A' = \left(\frac{81}{8}, \frac{93}{8}\right), \qquad B' = \left(-\frac{43}{40}, \frac{201}{40}\right), \qquad C' = \left(\frac{81}{8}, -\frac{27}{8}\right).$$

For example, $A - O = \left(-2, \frac{63}{8}\right)$ rotates to $\left(\frac{25}{8}, \frac{15}{2}\right)$, giving $A' = \left(\frac{81}{8}, \frac{93}{8}\right)$.

The hexagon $AA'CC'BB'$ is simple with these vertices in order, so the shoelace formula on $(5, 12), \left(\frac{81}{8}, \frac{93}{8}\right), (14, 0), \left(\frac{81}{8}, -\frac{27}{8}\right), (0, 0), \left(-\frac{43}{40}, \frac{201}{40}\right)$ gives area $\frac{1557}{10} = 155.7$. The closest integer is 156.

11. The integers from 1 to 64 are placed in some order into an 8×8 grid of cells with one number in each cell. Let $a_{i,j}$ be the number placed in the cell in row i and column j, and let M be the sum of the absolute differences between adjacent cells. That is,

$$M = \sum_{i=1}^{8} \sum_{j=1}^{7} \left(|a_{i,j+1} - a_{i,j}| + |a_{j+1,i} - a_{j,i}| \right).$$

Find the remainder when the maximum possible value of M is divided by 1000.

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Solution:

View the grid as a graph whose 112 edges join adjacent cells. Each edge contributes its larger endpoint value positively and its smaller one negatively, so $M = \sum_v c_v a_v$, where a_v is the entry in cell v and c_v is the number of neighbors of v with smaller entries minus the number with larger entries. Then $|c_v| \le \deg(v)$, which is 4 for the 36 interior cells, 3 for the 24 edge cells, and 2 for the 4 corners, and $\sum_v c_v = 0$ since each edge contributes $+1$ and -1.

Because $\sum_v c_v = 0$,

$$M = \sum_v c_v \left(a_v - \tfrac{65}{2} \right) \le \sum_v \deg(v) \left| a_v - \tfrac{65}{2} \right|.$$

By the rearrangement inequality this is maximized by pairing the 36 values farthest from $\frac{65}{2}$ (namely 1–18 and 47–64, whose deviations total 828) with the interior cells, the next 24 values (19–30 and 35–46, totaling 192) with the edge cells, and 31–34 (totaling 4) with the corners. Hence $M \le 4 \cdot 828 + 3 \cdot 192 + 2 \cdot 4 = 3896$.

Equality requires every cell holding a value at most 32 to be smaller than all its neighbors and every value at least 33 to be larger, which a checkerboard achieves: put 1–32 on the black cells (1–18 on interior blacks, 19–30 on edge blacks, 31–32 on black corners) and 33–64 on the white cells (33–34 on corners, 35–46 on edges, 47–64 in the interior). Every neighbor pair then compares white over black, so $M = 3896$, and the answer is $3896 \bmod 1000 = 896$.

12. Triangle $\triangle ABC$ lies in plane $\mathcal{P}$ with $AB = 6, AC = 4,$ and $\angle BAC = 90^\circ$. Let D be the reflection across $\overline{BC}$ of the centroid of $\triangle ABC$. Four spheres, all on the same side of $\mathcal{P}$, have radii $1, 2, 3,$ and r and are tangent to $\mathcal{P}$ at points $A, B, C,$ and D, respectively. The four spheres are also each tangent to a second plane $\mathcal{T}$ and are all on the same side of $\mathcal{T}$. The value of r can be written as $\frac{m}{n}$, where m and n are relatively prime positive integers. Find $m + n$.

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Solution:

A sphere of radius ρ tangent to $\mathcal{P}$ at P has center $P + \rho k$, where k is the upward unit normal of $\mathcal{P}$. Write $\mathcal{T}$ as $\{x : n \cdot x = c\}$ with unit normal $n = (n_1, n_2, h)$ in coordinates where $\mathcal{P}$ is the xy-plane. Tangency with all spheres on the same side means $n \cdot (P + \rho k) - c = \rho$ for each sphere, that is $n_1 x_P + n_2 y_P - c = (1 - h)\rho$. Here $h \neq 1$, since otherwise the left side would be constant while the radii differ. So $\rho = g(P)$ for the affine function $g(x, y) = \frac{n_1 x + n_2 y - c}{1 - h}$.

Take $A = (0, 0), B = (6, 0), C = (0, 4)$. The affine function with $g(A) = 1, g(B) = 2, g(C) = 3$ is $g(x, y) = 1 + \frac{x}{6} + \frac{y}{2}$. The centroid is $G = \left(2, \frac{4}{3}\right)$, and line BC is $2x + 3y = 12$. Since $2 \cdot 2 + 3 \cdot \frac{4}{3} - 12 = -4$, reflecting gives

$$D = G + \frac{8}{13}(2, 3) = \left(\frac{42}{13}, \frac{124}{39}\right).$$

Therefore $r = g(D) = 1 + \frac{1}{6} \cdot \frac{42}{13} + \frac{1}{2} \cdot \frac{124}{39} = 1 + \frac{21}{39} + \frac{62}{39} = \frac{122}{39}$. (Such a plane exists: the normal condition $(1 - h)^2 |\nabla g|^2 = 1 - h^2$ with $|\nabla g|^2 = \frac{5}{18}$ gives $h = -\frac{13}{23}$.) Since $\gcd(122, 39) = 1$, the answer is $122 + 39 = 161$.

13. For each nonnegative integer r less than 502 define

$$S_r = \sum_{m \geq 0} \binom{10{,}000}{502m + r},$$

where $\binom{10{,}000}{n}$ is defined to be 0 when $n > 10{,}000$. That is, S_r is the sum of all the binomial coefficients of the form $\binom{10{,}000}{k}$ for which $0 \leq k \leq 10{,}000$ and $k - r$ is a multiple of 502.

Find the number of integers in the list $S_0, S_1, S_2, \ldots, S_{501}$ that are multiples of the prime number 503.

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Solution:

Work in the ring $\mathbb{F}_{503}[x]/(x^{502} - 1)$. Reducing $(1+x)^{10000} = \sum_k \binom{10000}{k} x^k$ replaces each exponent k by $k \bmod 502$, so

$$(1+x)^{10000} \equiv \sum_{r=0}^{501} S_r\, x^r \pmod{503,\ x^{502} - 1}.$$

Since 503 is prime, $(1+x)^{503} \equiv 1 + x^{503} \pmod{503}$, and $x^{503} = x \cdot x^{502} \equiv x$, so $(1+x)^{503} \equiv 1 + x$ in this ring. Writing $10000 = 19 \cdot 503 + 443$,

$$(1+x)^{10000} = \left((1+x)^{503}\right)^{19} (1+x)^{443} \equiv (1+x)^{19} (1+x)^{443} = (1+x)^{462}.$$

As $462 < 502$, no exponents fold, so $S_r \equiv \binom{462}{r} \pmod{503}$ for $0 \leq r \leq 501$, where $\binom{462}{r} = 0$ for $r > 462$.

For $0 \leq r \leq 462$ the binomial coefficient $\binom{462}{r}$ is not divisible by $503:$ both 462 and r are single digits in base 503, so Lucas' theorem gives a nonzero value (indeed $\binom{462}{r} = \frac{462!}{r!\,(462-r)!}$ involves no factor of 503). Hence $S_r \equiv 0 \pmod{503}$ exactly for $r = 463, 464, \ldots, 501$, which is $501 - 463 + 1 = 39$ values.

14. In an equiangular pentagon, the sum of the squares of the side lengths equals 308, and the sum of the squares of the diagonal lengths equals 800. The square of the perimeter of the pentagon can be expressed as $m\sqrt{n}$, where m and n are positive integers and n is not divisible by the square of any prime. Find $m+n$.

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Solution:

In an equiangular pentagon each side direction turns by the exterior angle 72°, so the sides are the vectors $s_k u_k$ for $k=1,\dots,5$, where $u_k=(\cos 72k^\circ, \sin 72k^\circ)$ and $\sum_k s_k u_k=0$. Write $Q=\sum s_k^2=308$, $P_1=\sum_k s_k s_{k+1}$, and $P_2=\sum_k s_k s_{k+2}$ (indices cyclic). Each diagonal is a sum of two consecutive side vectors, so its square is $s_{k+1}^2+s_{k+2}^2+2s_{k+1}s_{k+2}\cos 72^\circ$, and summing all five gives

$$800=2Q+2\cos 72^\circ\, P_1, \qquad \text{so} \qquad 2\cos 72^\circ\, P_1=800-616=184.$$

Expanding $\left|\sum_k s_k u_k\right|^2=0$, the angle between u_k and u_{k+1} is 72° and between u_k and u_{k+2} is 144° :

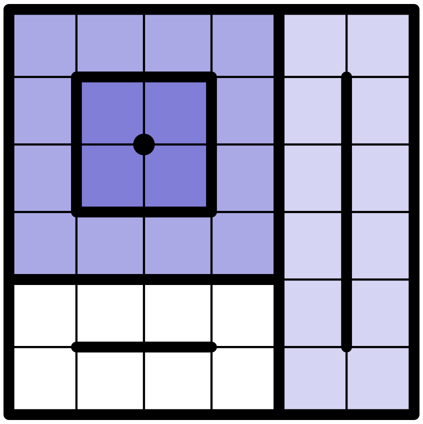
$$0=Q+2\cos 72^\circ\, P_1+2\cos 144^\circ\, P_2=308+184-2\cos 36^\circ\, P_2,$$

so $2\cos 36^\circ\, P_2=492$. Using $\cos 72^\circ=\frac{\sqrt{5}-1}{4}$ and $\cos 36^\circ=\frac{\sqrt{5}+1}{4}$, we get $2P_1=\frac{184}{\cos 72^\circ}=184\left(\sqrt{5}+1\right)$ and $2P_2=\frac{492}{\cos 36^\circ}=492\left(\sqrt{5}-1\right)$.
The square of the perimeter is

$$\left(\sum s_k\right)^2=Q+2P_1+2P_2=308+184\sqrt{5}+184+492\sqrt{5}-492=676\sqrt{5}.$$

Therefore $m+n=676+5=681.$

15. Let $a, b,$ and n be positive integers with both a and b greater than or equal to 2 and less than or equal to $2n$. Define an $a \times b$ *cell loop* in a $2n \times 2n$ grid of cells to be the $2a + 2b - 4$ cells that surround an $(a-2) \times (b-2)$ (possibly empty) rectangle of cells in the grid. For example, the following diagram shows a way to partition a 6×6 grid of cells into 4 cell loops.



Find the number of ways to partition a 10×10 grid of cells into 5 cell loops so that every cell of the grid belongs to exactly one cell loop.

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Solution:

Since the five loops cover $\sum(2(a_i + b_i) - 4) = 100$ cells, $\sum(a_i + b_i) = 60$. Every loop has an even number of cells, so no odd-by-odd rectangle can be exactly filled by loops; and filling a rectangle whose shortest even side is e requires at least $\frac{e}{2}$ loops, since peeling off an outermost loop shrinks that side by exactly 2 while splitting a rectangle into smaller ones only adds up such requirements. Now consider the *outermost* loops of a partition (those whose rectangles lie inside no other loop's rectangle): their rectangles tile the 10×10 square. If outermost rectangle R_i has shortest even side e_i, it uses $n_i \geq \frac{e_i}{2}$ loops and covers at most $10\, e_i$ cells. Summing over the tiling, $100 \leq \sum 10\, e_i \leq 20 \sum n_i = 100$, so equality holds throughout: each R_i spans the full 10 in one direction, has even width e_i, and is filled with exactly $\frac{e_i}{2}$ loops. Two full-length slabs in different directions would overlap, so the outermost rectangles are the whole square or parallel slabs, and the same equality argument repeats inside every loop's inner rectangle.

Let $s(w)$ be the number of ways to fill a full-height slab of even width w with $\frac{w}{2}$ loops. A width-2 slab is a single loop: $s(2) = 1$. A width-4 slab is a 10×4 loop around an 8×2 loop: $s(4) = 1$. A width-6 slab is a 10×6 loop around an 8×4 region holding two loops — either nested (8×4 around 6×2) or two 8×2 slabs — so $s(6) = 2$. A width-8 slab surrounds an 8×6 region holding three loops: an 8×6 loop around a 6×4 region with two loops (2 ways as before), or full-height strips of widths $2 + 2 + 2$ (1 way), or widths $2 + 4$ in two orders (2 ways), so $s(8) = 5$. The same recursion counts the full square: a 10×10 loop around an 8×8 region with four loops, where the $4 \times 4, 6 \times 6$, and 8×8 regions admit 3, then $3 + 3 + 3 = 9$, then $9 + 9 + 9 = 27$ fillings (single nested loop, vertical strips, or horizontal strips at each stage).

Finally, tally the outermost structures. The single 10×10 rectangle gives 27 partitions. For parallel slabs, the widths form a composition of 10 into even parts with at least two parts, and orientations (vertical or horizontal) double the count: $(2, 2, 2, 2, 2)$ gives 1; $(4, 2, 2, 2)$ in 4 orders gives 4; $(4, 4, 2)$ in 3 orders gives 3; $(6, 2, 2)$ in 3 orders gives $3 \cdot 2 = 6$; $(6, 4)$ in 2 orders gives $2 \cdot 2 = 4$; and $(8, 2)$ in 2 orders gives $2 \cdot 5 = 10$, for 28 per orientation. The total is $27 + 2 \cdot 28 = 83$.

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